

## Problem Set 4 (10 points)

In international security, the value of alliances depends on the reliability of the allies that form them. The anticipation that a reliable ally will defend a target of aggression may lead an attacker to anticipate punishment. Anticipating punishment, the attacker will not attack. The alliance game in Figure 1 attempts to model this reasoning. A challenger (Nation A) has the choice to attack a target (Nation B) or not to attack it and its optimal action hinges on its beliefs about the target's ally (Nation C). Crucially, the ally may be reliable or unreliable and only the ally knows if it is reliable; the challenger is not sure. If the ally is (perceived to be) reliable, then the challenger will not attack, while if the ally is (perceived to be) unreliable, the challenger will attack. If the challenger attacks and it is right in the estimate of the ally's reliability, then the ally will stay out.

In Figure 1, there are three players: Country A, the potential attacker; country B, the potential target; and country C, the potential ally of country B. For each outcome, A's payoff is listed first, B's payoff is listed second, and C's payoff is listed third. Preferences are ordered so that  $a$  is the most preferred outcome, while  $d$  is the worst. That is,  $a > b > c > d$ .

C might be one of two types of allies, a *reliable* or an *unreliable* ally. The top subgame of the tree represents an ally C that is reliable. Given the move by nature, there is a probability  $p$  that the attacker is facing a target with a reliable ally. The lower subgame of the tree represents an ally C that is unreliable. Given the move by nature, there is a probability  $1 - p$  that the attacker is facing a target with an unreliable ally. The potential outcomes of the game are the "Status Quo", "A Wins", a war where "B Fights Alone", and a war where "B and C Join against A."

First, solve each part of the game as if there were no uncertainty. Write out the preference orderings for the players. The payoffs are actually already written in the game tree in Figure 1. For reference, write them out here in summary form. For example, working from the game we can see that A's preferences are

$$\text{A Wins} \succ \text{B Fights Alone} \succ \text{Status Quo} \succ \text{War against B and C}$$

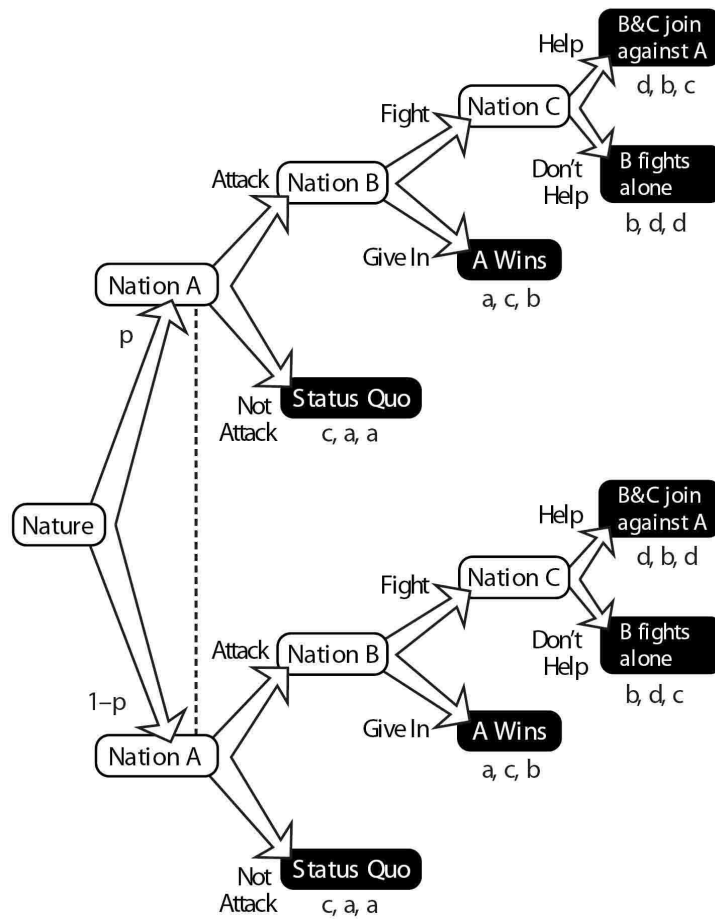


Figure 1: Alliance Game

(Hint: As an exercise or if it helps, replace the payoffs  $a$ ,  $b$ ,  $c$ , and  $d$  in Figure 1 with numeric payoffs 4, 3, 2, 1. But ultimately, submit your answers using the payoffs  $a$ ,  $b$ ,  $c$ , and  $d$ )

- In a similar way, write out B's preferences, C's preferences if C is a reliable ally, and C's preferences if C is an unreliable ally (1 point).
- Solve the upper and lower subgames of the alliance reliability game in Figure 1 by backward induction (2 points).
- What is the outcome if the challenger knows that the ally is reliable (1 point)?

- d. What is the outcome if the challenger knows that the ally is unreliable (1 point)?

Second, solve for the critical value of probability  $p$ . Because the outcome in the two subtrees is different, what the challenger does is dependent on the challenger's beliefs about the ally's type.

- f. Compute the expected utility to the challenger from attacking. Write out the challenger's  $EU_C(\text{attack})$  and simplify the expression as much as you can (1 point).
- g. Compute the expected utility to the challenger from not attacking. Write out the challenger's  $EU_C(\text{do not attack})$  and simplify the expression as much as you can (1 point).
- h. To attack, the payoff from the attack to the challenger must be greater than the payoff from the status quo (not attacking). Solve for the critical value of  $p$  that makes this true. (Hint: the result in this case will not be a number but an expression involving payoffs  $a$ ,  $b$ ,  $c$ , and/or  $d$ .) (3 points)